

Derivacije

1. Derivirajmo sljedeću funkciju primjenjujući pravilo za zbrajanje/oduzimanje:

a) $f(x) = 3x^3 - 2x^2 + 3x - 1$

$$f'(x) = (3x^3)' - (2x^2)' + (3x)' - 1'$$

$$f'(x) = 3 \cdot 3x^{3-1} - 2 \cdot 2x^{2-1} + 3 \cdot 1 - 0$$

$$f'(x) = 9x^2 - 4x + 3$$

$$(I \pm II)' = I' \pm II'$$

$$c' = 0$$

$$x' = 1$$

$$(x^n)' = n \cdot x^{n-1}$$

2. Derivirajmo sljedeće funkcije primjenjujući pravilo za množenje/dijeljenje:

a) $f(x) = \underbrace{(3-4x)}_I \cdot \underbrace{(x^2-3x+1)}_{II}$

$$f'(x) = (3-4x)' \cdot (x^2-3x+1) + (3-4x) \cdot (x^2-3x+1)'$$

$$f'(x) = (0-4)(x^2-3x+1) + (3-4x)(2x-3)$$

$$f'(x) = -4x^2 + 12x - 4 + 6x - 8x^2 - 9 + 12x$$

$$f'(x) = -12x^2 + 30x - 13$$

$$(I \cdot II)' = I' \cdot II + I \cdot II'$$

b) $f(x) = \frac{\underbrace{3x+4}_I}{\underbrace{2x-1}_{II}}$

$$f'(x) = \frac{(3x+4)' \cdot (2x-1) - (3x+4) \cdot (2x-1)'}{(2x-1)^2}$$

$$f'(x) = \frac{3(2x-1) - 2(3x+4)}{(2x-1)^2}$$

$$f'(x) = \frac{6x-3-6x-8}{(2x-1)^2}$$

$$f'(x) = -\frac{11}{(2x-1)^2}$$

$$\left(\frac{I}{II}\right)' = \frac{I' \cdot II - I \cdot II'}{II^2}$$

3. Derivirajmo sljedeće složene funkcije:

a) $f(x) = (5x^2 - 3x + 1)^2$ ↗ n
f

$$(f^n)' = n \cdot f^{n-1} \cdot f'$$

$$f'(x) = 2 \cdot (5x^2 - 3x + 1)^{2-1} \cdot (5x^2 - 3x + 1)'$$

$$f'(x) = 2 \cdot (5x^2 - 3x + 1) \cdot (10x - 3)$$

b) $f(x) = e^{-x^2+2}$ ↗ f

$$(e^f)' = e^f \cdot f'$$

$$f'(x) = e^{-x^2+2} \cdot (-x^2 + 2)'$$

$$f'(x) = e^{-x^2+2} \cdot (-2x)$$

$$f'(x) = -2x \cdot e^{-x^2+2}$$

c) $f(x) = \ln(e^x + 1)$ ↗ f

$$(\ln f)' = \frac{1}{f} \cdot f'$$

$$f'(x) = \frac{1}{e^x + 1} \cdot (e^x + 1)'$$

$$f'(x) = \frac{1}{e^x + 1} \cdot e^x$$

$$f'(x) = \frac{e^x}{e^x + 1}$$